

Dimensional analysis of yielding and pounding structures for records without distinct pulses

Elias Dimitrakopoulos^{a,*}, Andreas J. Kappos^a, Nicos Makris^b

^a Department of Civil Engineering, Aristotle University of Thessaloniki, GR 54124, Greece

^b Department of Civil Engineering, University of Patras, GR 26500, Greece

ARTICLE INFO

Article history:

Received 20 November 2008

Received in revised form

17 February 2009

Accepted 19 February 2009

Keywords:

Non-linear structural dynamics

Dimensional analysis

Physical similarity

Earthquake engineering

ABSTRACT

The seismic response of two fundamental mechanical configurations of earthquake engineering, the elastic–plastic system and the pounding oscillator, is revisited with the aid of dimensional analysis. Starting from the previous work of the authors which focused on pulse-type excitations, the paper offers an alternative, yet physically motivated, way to present the response of yielding and pounding structures under excitations with arbitrary time history. It is shown, that when the appropriate time and length scales are adopted, dimensional analysis can be implemented and remarkable order emerges in the response. Regardless of the acceleration level and frequency content of the excitation, all response spectra become self-similar and when expressed in dimensionless terms, resulting from dimensional analysis, follow a single master curve. The study proposes such scales together with the associated selection criteria among the available in literature strong ground motion parameters and shows that the proposed approach reduces drastically the scatter in the response.

© 2009 Elsevier Ltd. All rights reserved.

1. Introduction

A major challenge in studying the response of structures with non-linear behaviour, is no longer the dynamic response analysis of a specific configuration, but rather the interpretation and presentation of the response analysis in a way that is most meaningful for a broad class of structural configurations. This challenge emerged partly due to: (1) the ability of modern computers to rapidly produce a large number of non-linear solutions, (2) an ever-increasing database of recorded ground motions with quite complex patterns, and (3) the wide scatter in the structural response calculated from time-history analysis using historic records.

In an effort to partly address this challenge Makris and Black [1,2] revisited the inelastic response of elastic–plastic and bilinear systems with the aid of dimensional analysis [3–5] and subsequently demonstrated the applicability and promise of the proposed approach to various structural frames known in the literature [5]. More recently, the response of pounding oscillators, both elastic and inelastic, was also revisited with formal dimensional analysis by Dimitrakopoulos et al. [6,7]. In these studies it was shown that the non-linear response of yielding, as well as pounding, structures, under pulse-type excitations, becomes self-similar when expressed in the appropriate dimensionless

Π -terms derived from formal dimensional analysis. Self-similarity is a special type of symmetry which is invariant with respect to scale transformations and it is of unique importance when ordering non-linear behaviour. It is interesting to note, that self-similarity prevails even when the two non-linearities, the inelastic behaviour of the structure and the boundary non-linearity of the pounding phenomenon, coexist [6].

The implementation of dimensional analysis, in the aforementioned studies of Makris et al. [1,2,5] and Dimitrakopoulos et al. [6,7], hinges upon the existence of a distinct time scale and a length scale that characterize the most energetic component of the ground shaking. In the case of records with distinct pulses, these scales emerge naturally from the acceleration amplitude, α_p , and the duration of the pulse, T_p , and can be formally extracted with validated mathematical models available in the literature e.g. [8]. The aforementioned studies brought forward the profound significance of the energetic length scale of the pulse: $L_e = \alpha_p / \omega_p^2 = (1/4\pi^2) \alpha_p T_p^2$ coined by Makris [1], which liberates the non-linear response from the need to refer to a ‘substitute’ system (e.g. the elastic SDOF oscillator) and reveals the property of self-similarity in the response.

The aforementioned work of the authors on the dimensional analysis of non-linear systems was justified by the distinct coherent pulses identified in a wide class of strong motion records, mostly near source ones. The present study is motivated by the need to extend the application of the dimensional analysis approach for excitations without distinct pulses, as well as to contribute to the goals of the work of Kappos and Kyriakakis [9]

* Corresponding author.

E-mail address: elias.dimitrakopoulos@gmail.com (E. Dimitrakopoulos).

who re-evaluated the problem of reducing the scatter in the structural response to natural records.

2. Time and length scales in earthquake records with distinct pulses

The present section builds on the work of several researchers [8,10,11] who adopted typical pulses in order to simulate (the long period component of) real earthquake excitations.

During the last three decades, an ever-increasing database of recorded ground motions has shown that the kinematic characteristics of the ground motion near the fault of major earthquakes contains distinguishable acceleration and velocity pulses (e.g. Fig. 1). The relatively simple form and the destructive potential of near source ground motions have motivated the development of various closed-form expressions which approximate their leading kinematic characteristics. Physically realizable trigonometric pulses can adequately describe the impulsive

character of near-fault ground motions both qualitatively and quantitatively.

Fig. 1 plots the acceleration, A_g , velocity, V_g , and displacement, D_g , time-histories of the NS record, from the 1992 Erzincan earthquake. It is clear that: (a) the main characteristics of the record can be captured with the use of a simple pulse, and (b) that the period of the distinct pulse, corresponds quite well to the maximum peak of the Fourier spectrum. Fig. 2 plots the time-histories and the Fourier spectrum of the A399 record from the Athens 1999 earthquake. Unlike the Erzincan record, the period of the distinct pulse does not correspond to the peak of the Fourier amplitude spectrum, yet it is still distinguishable in it.

Fig. 3 presents the Newhall station record, from the 1994 Northridge earthquake. At least four of the non-high-frequency peaks of the Fourier spectrum ($T_1 = 0.59$ s, $T_2 = 0.71$ s, $T_3 = 1.3$ s and $T_4 = 2.0$ s) can be corresponded to pertinent distinct pulses.

Distinct pulses appear usually, but not always [12,13], in near-source ground motions (Fig. 1). On the other hand, there exist far-field records that contain also distinguishable pulses; a typical

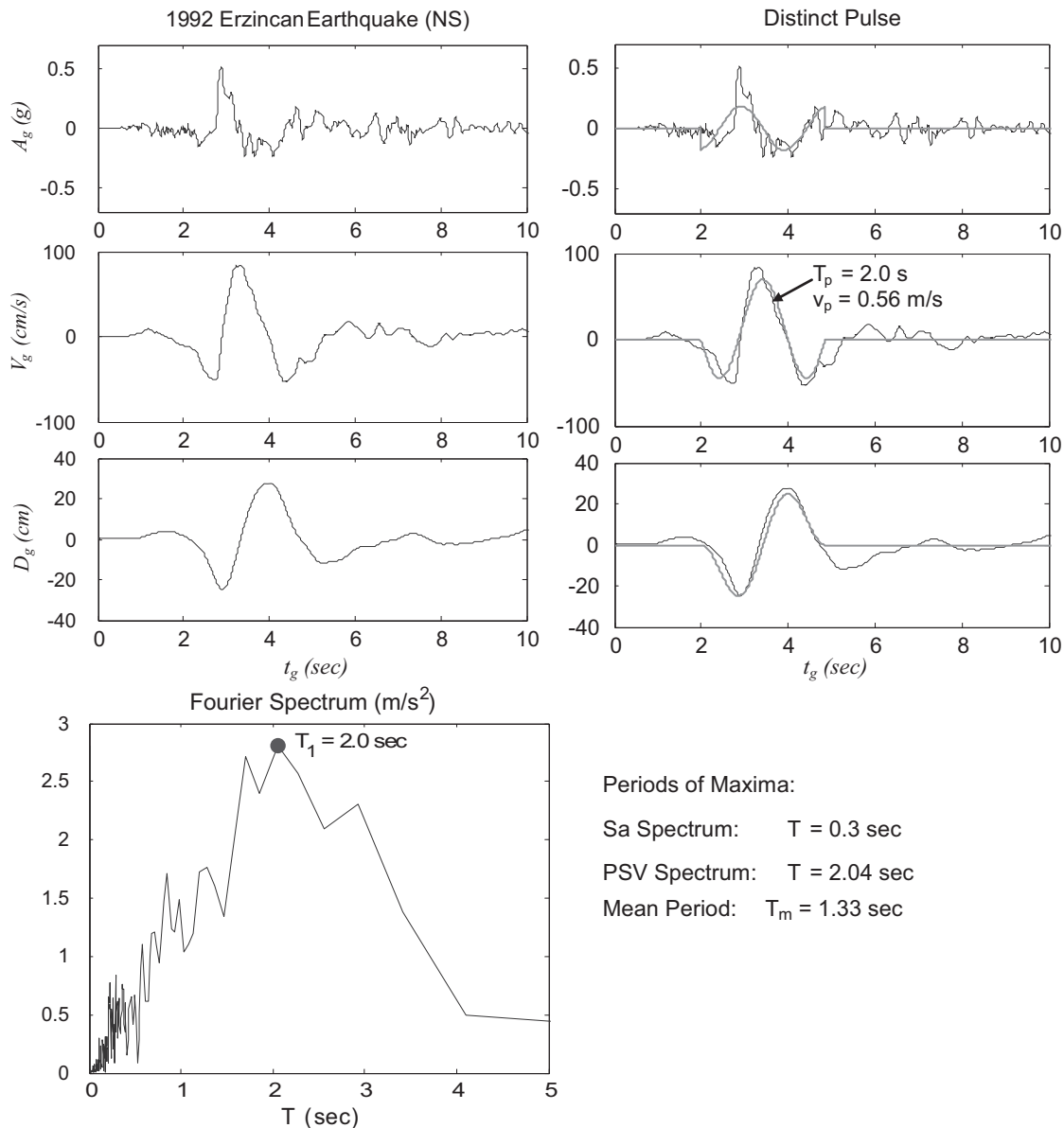


Fig. 1. Top left: acceleration, A_g , velocity, V_g , and displacement, D_g , time-histories of the Erzincan (NS) earthquake record. Top right: the distinct pulse. Bottom: the Fourier amplitude spectrum.

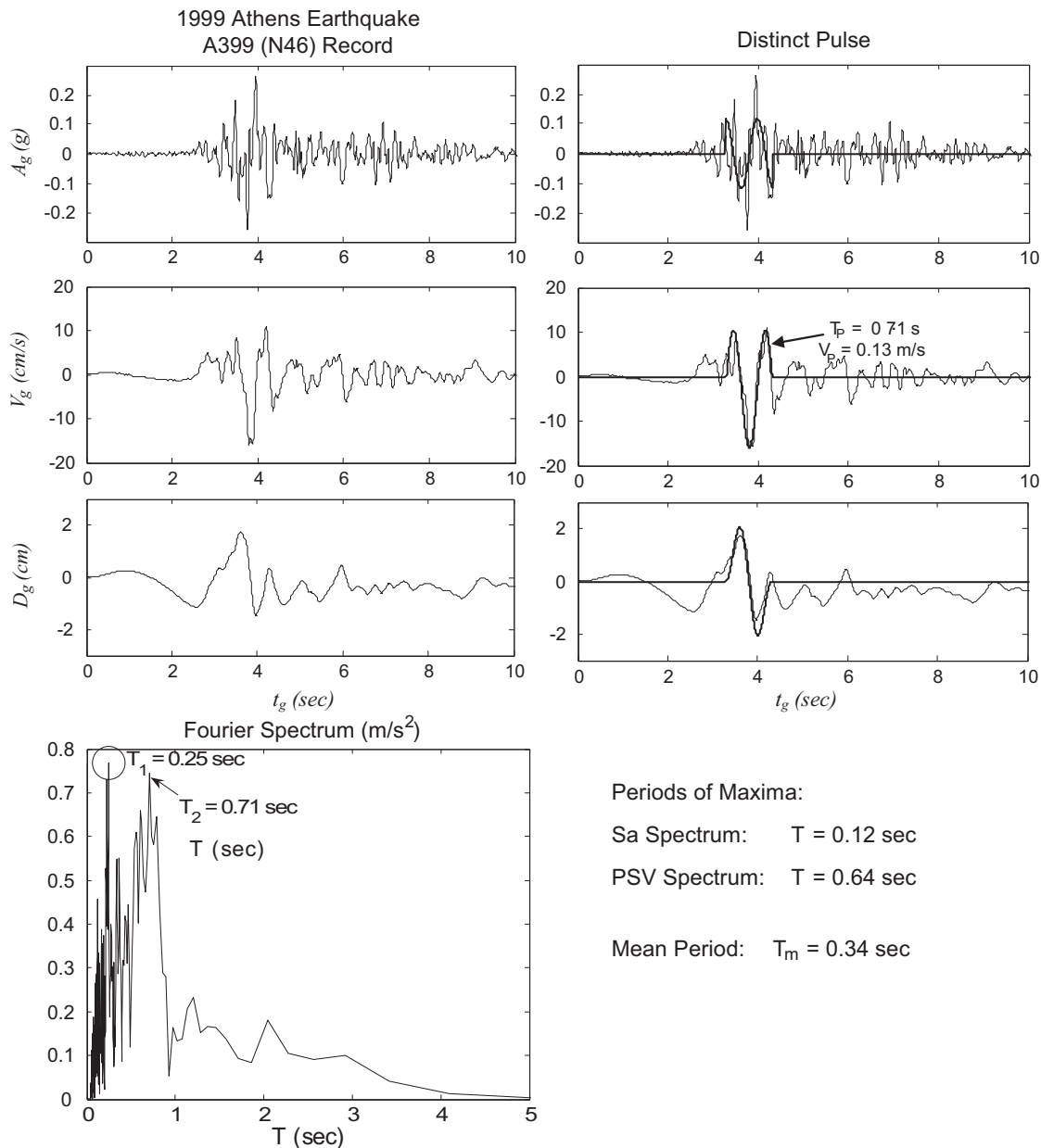


Fig. 2. Top left: acceleration, A_g , velocity, V_g , and displacement, D_g , time-histories of the A399 (N46) record. Top right: the distinct pulse. Bottom: the Fourier amplitude spectrum.

example is the Bucharest record of the 1977 Vrancea earthquake (Fig. 4) where the presence of a deposit filtered/amplified an earthquake signal which originated more than 160 km away [14].

The present analysis confirms that in the case of records with distinct pulses, the period of these pulses in general corresponds to visible peaks in the associated Fourier spectrum. Thus, the Fourier spectrum can be exploited in order to identify the time scale of such excitations, provided the distinguishable pulses do not correspond to high-frequency motions. This observation is in agreement with the model of Mavroeidis and Papageorgiou [8], according to which the (time scale) 'duration of the pulse', T_p , is considered as $T_p = 1/f_p$, where f_p is the predominant frequency of the signal. It is recalled that, even though the duration of a pulse usually coincides with its predominant period, the two notions 'duration' and 'period', are differentiated for excitations with more than one loading cycle.

To summarize, for records with (one or more) distinct pulses, it is usually feasible to identify the period of the non-high-frequency

pulses in the pertinent Fourier spectrum. This observation brings forward the essential time scale of an excitation when a peak response parameter is of interest, i.e. its frequency content, and paves the way for the generalization of the dimensional response analysis approach, for records without distinct pulses.

3. Time and length scales in earthquake records without distinct pulses

Several challenges emerge when trying to generalize the concept of using typical pulses in order to simulate a real record. First of all, in many records there is no distinct pulse, but rather their form is composed of random acceleration spikes (Fig. 5). On the other hand, even when a record contains more than one distinguishable pulse, it is not clear how to incorporate the characteristics of those pulses into a single time scale (or parameter). Hence arises the need to characterize a record with

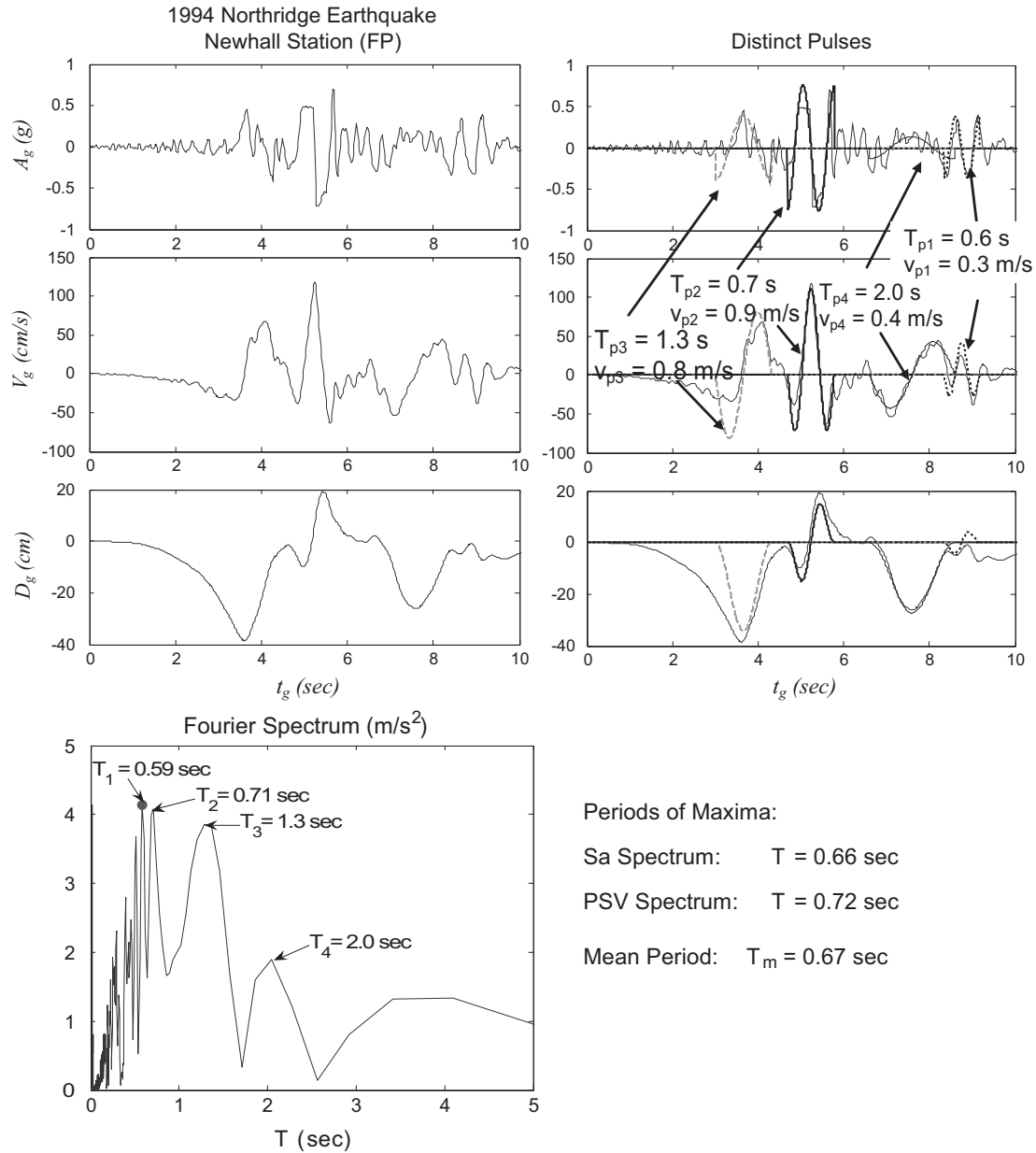


Fig. 3. Top left: acceleration, A_g , velocity, V_g , and displacement, D_g , time-histories of the Newhall station record. Top right: the distinct pulse. Bottom: the Fourier amplitude spectrum.

a time scale directly, without comparing it with one or more typical pulses.

The present paper proposes the use of appropriate strong ground motion parameters, already available in the literature, which allow the implementation of the dimensional analysis approach in the case of records with arbitrary shape. Building on the previous work on the dimensional response analysis of structures [1,2,6,7] the desired properties, of the appropriate time and length scales of an excitation that bring forward the property of self-similarity, can be defined.

3.1. Criteria for selection of the strong ground motion parameters

1. The appropriate strong ground motion parameters should be independent of the structural response. This is essential, since in order to reveal the property of self-similarity, the non-linear response must be liberated from the need to refer to a

'substitute system' (such as an equivalent linear-elastic system). This was illustrated for both yielding [1,2] and pounding [6,7] structures.

2. Given that the response function has to be dimensionally homogeneous, two reference dimensions are needed, that of time $[T]$ and length $[L]$. By analogy to the time and length scales of typical pulses, the desired strong ground motion parameters should be two: one with dimensions of time, and one length scale with dimensions of velocity or acceleration. Hence, a pair of parameters with appropriate dimensions is requested, such that all involved quantities can be scaled with the same two parameters and not with a single one.
3. For practical reasons, it is convenient to use a length scale/parameter that is linearly dependent on the amplitude of the excitation.
4. If feasible, the adopted parameters should have an unambiguous physical meaning and be easily estimated.

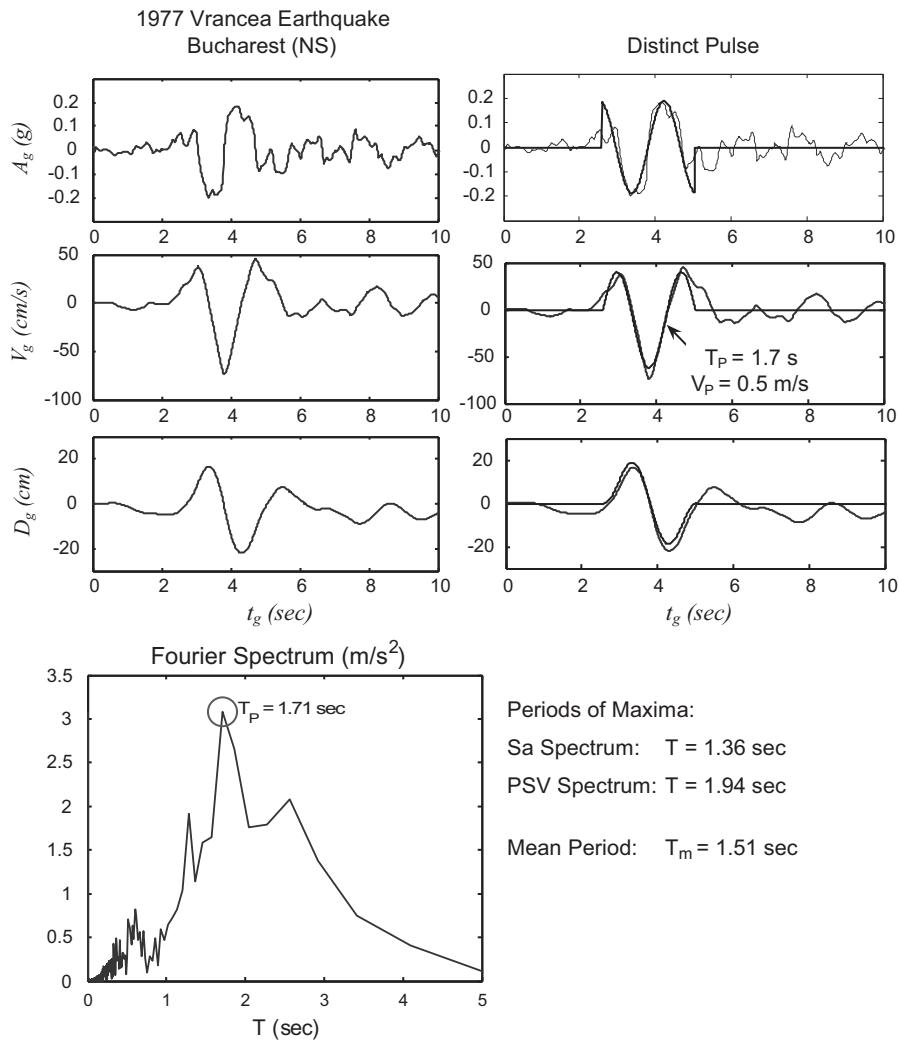


Fig. 4. Top left: acceleration, A_g , velocity, V_g , and displacement, D_g , time-histories of the Bucharest (NS) record. Top right: the distinct pulse. Bottom: the Fourier amplitude spectrum.

3.2. Proposed strong ground motion parameters

The aforementioned desired properties comprise the criteria based on which the appropriate strong ground motion parameters (one time scale and one length scale) can be selected among the available relevant parameters in the literature.

Hancock and Bommer, after reviewing the pertinent literature [15], and evaluating the results of an analytical study [16], confirmed that the duration of an excitation has no influence when a peak response quantity is of interest, like the maximum response displacement, velocity or acceleration. Hancock and Bommer's recent conclusions regarding the duration of an excitation are in agreement with earlier findings by Kappos [17]. Since the present study focuses on peak response parameters, the appropriate time scale should describe the frequency content of the excitation, not its duration. Mylonakis and Voyagaki [18] considered pulse-type excitations and showed that the maximum ductility of an inelastic oscillator is much more sensitive to the time-history shape of an excitation than to the number of loading cycles (of the same excitation).

The period corresponding to the peak acceleration of the elastic response spectrum, T_{PSA} , is probably the most extensively used frequency parameter in earthquake engineering. Yet, it is

well known that two acceleration spectra with the same T_{PSA} may differ substantially in shape [19]. Similarly, the period of the peak velocity of the elastic response spectrum, T_{PSV} , is known to be an effective parameter for characterizing the frequency content of an excitation [18,20]. However, T_{PSA} , T_{PSV} , as well as the period of any other peak spectral quantity, are (structural) response-dependent parameters and hence are not considered further herein, since they contradict the first selection criterion in Section 3.1.

On the contrary, the period of the maximum Fourier amplitude spectrum is not dependent on the structural response. It is well known though, that the Fourier amplitude spectrum may exhibit a peak at high frequencies, while the most energetic components of the excitation correspond to lower frequencies (see e.g. Fig. 2). A simple frequency parameter that overcomes this shortcoming and encloses information about the shape of the Fourier spectrum is the mean period, T_m , proposed by Rathje et al. [19]. The mean period, T_m , averages the periods of the Fourier spectrum according to the equation

$$T_m = \frac{\sum_i C_i^2 (1/f_i)}{\sum_i C_i^2} \quad (1)$$

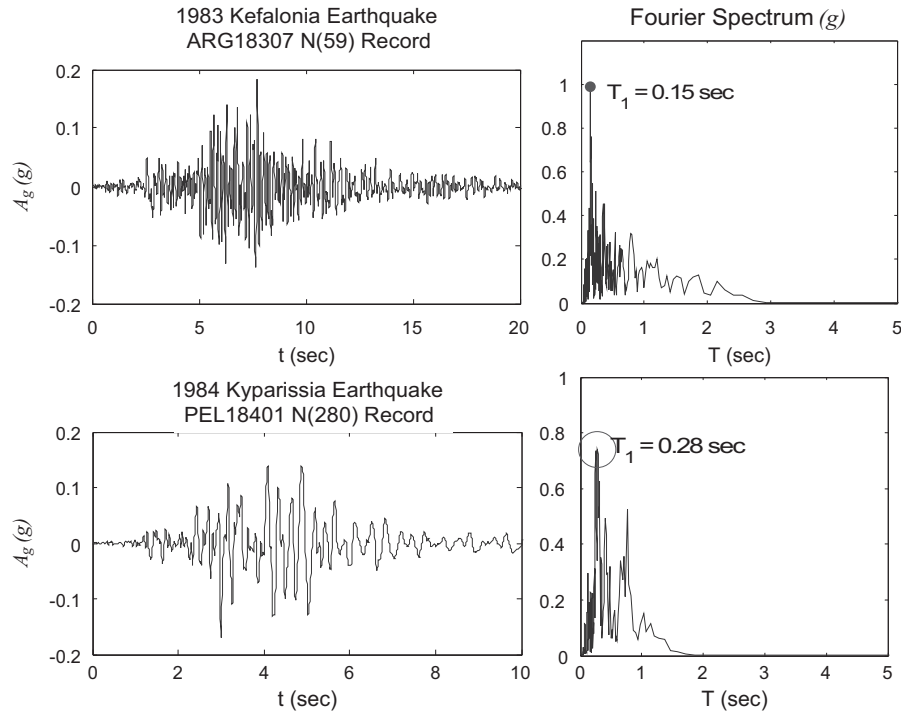


Fig. 5. Left: acceleration time-histories of typical (Greek) records without distinct pulse. Right: the pertinent Fourier amplitude spectra.

where C_i are the Fourier amplitudes of the accelerogram and f_i the discrete Fourier transform frequencies between 0.25 and 20 Hz. The mean period is known to perform equally well as the smoothed spectral predominant period [19]. Unlike the pertinent spectral periods, T_m does not engage structural response and hence complies with the first desired property.

Concerning the appropriate length scale, the spectral intensity, SI , either in the original form proposed by Housner [21] or in the modified one proposed by Kappos [17,22] is an effective strong ground motion parameter that has been extensively used by many researchers [9,23]. However, SI is also a response-dependent parameter like the T_{PSA} , T_{PSV} and for the same reason will not be further considered.

Kappos and Kyriakakis [9] re-evaluated the ability of the most commonly used scaling parameters to reduce the scatter in the analytically estimated structural response. They concluded that the acceleration-based parameters, such as the peak ground acceleration, PGA , the Arias intensity, AI , and the root-mean-square acceleration, $RMSA$, describe better the response of intermediate and long period structures, in contrast to velocity-based parameters, such as the peak ground velocity, PGV , the spectrum intensity, SI , and the Fajfar [24] index, I_v , that describe better the response of intermediate and long period structures. The conclusions of Kappos and Kyriakakis [9] were further confirmed by the comparative study of an even wider set of strong ground motion parameters conducted by Riddell [25]. Moreover, Makris and Black [26] considered both linear and non-linear structural responses under pulse-type excitations and concluded for the set of records that they examined that PGA is a more representative intensity measure than the PGV of the earthquake shaking.

Another interesting strong ground motion parameter is the cumulative absolute velocity, CAV , which equals the area under the (absolute) acceleration versus duration curve. Equivalently, it can be considered as the sum of the consecutive peak-to-valley

distances in the velocity time history. Thus, CAV condenses information for the amplitude and the duration of motion and has been known to express the destructive potential of the excitation [27]. The disadvantage of cumulative parameters like CAV is their dependency on the duration of the excitation regarding which there is still great uncertainty [15].

Also, more complex parameters like the Fajfar index, I_v , or those examined in references [28,29] are not considered herein, either because their dimensions are not the sought ones, or because they do not comprise a pair of parameters (2nd desired property).

Based on the preceding discussion, the peak ground acceleration, PGA , and the mean period, T_m [19] are proposed as the appropriate length and time scales, respectively, of an excitation with arbitrary time-history shape.

3.3. Selected ground motions from previous earthquakes

The ground motions used in the present study comprise most of the available historic Greek records with pga of 0.1 g or more [30]. It is reminded that in Greece all types of faults coexist and it should be noted that no shape restriction (pulse-type or otherwise) has been applied for the records examined. Table 1 lists these records together with their pertinent characteristics; R stands for the epicentral distance and f_c for the cut-off frequency.

The first set of records (Table 1, records: 1–62) consists of all Greek digital records used in [30] and from the analogue ones, those that have been filtered with a frequency lower than 0.25 Hz; it should be noted that all records used herein have been re-processed (Margaris, V., private communication, 2008) with cut-off frequencies generally lower than those in [30]. The pertinent, elastic and inelastic, spectra of these excitations can be found in the recent paper of Karakostas et al. [30]. Also, a second set of Greek records (essentially a subset of the previous one) has been

Table 1
Main characteristics of Greek records used [29].

| No. | Earthquake/date site | Name | Component | M_w | R (km) | Soil | f_c (Hz) | PGA (cm/s ²) | T_m (s) |
|-----|----------------------|----------|-----------|-------|--------|------|------------|--------------------------|-----------|
| 1* | Volvi, 20.6.78 | THEA7802 | L | 6.5 | 26 | C | 0.2 | 142.56 | 0.5 |
| 2* | (Thessaloniki) | THEA7802 | T | 6.5 | 26 | | 0.2 | 146.94 | 0.47 |
| 3* | Alkyonides, 24.2.81 | KORA8101 | L | 6.7 | 32 | C | 0.15 | 229.5 | 0.68 |
| 4* | (Corinth) | KORA8101 | T | 6.7 | 32 | | 0.15 | 274.41 | 0.55 |
| 5* | Kefalonia, 17.1.83 | ARG18301 | L | 7 | 35 | B | 0.25 | 173.31 | 0.26 |
| 6* | (Argostoli) | ARG18301 | T | 7 | 35 | | 0.25 | 142.5 | 0.33 |
| 7* | Kefalonia, 23.3.83 | ARG18307 | L | 6.2 | 26 | | 0.25 | 179.81 | 0.2 |
| 8* | (Argostoli) | ARG18307 | T | 6.2 | 26 | | 0.25 | 219.2 | 0.23 |
| 9* | Kyparissia, 25.10.84 | PEL18401 | L | 5 | 9 | A | 0.2 | 166.63 | 0.29 |
| 10* | (Pelekanada) | PEL18401 | T | 5 | 9 | | 0.2 | 172.75 | 0.31 |
| 11* | Kalamata, 13.9.86 | KAL18601 | L | 6 | 12 | B | 0.1 | 229.26 | 0.6 |
| 12* | (Kalamata) | KAL18601 | T | 6 | 12 | | 0.1 | 263.88 | 0.53 |
| 13 | Kalamata, 15.9.86 | KAL18608 | L | 5.3 | 3 | | 0.15 | 232.79 | 0.51 |
| 14 | (Kalamata) | KAL18608 | T | 5.3 | 3 | | 0.15 | 137.11 | 0.46 |
| 15 | | KAL28602 | T | 5.3 | 3 | | 0.1 | 254.31 | 0.55 |
| 16 | Zakynthos, 16.10.88 | ZAK18804 | L | 6 | 20 | C | 0.2 | 133.02 | 0.42 |
| 17 | | ZAK18804 | T | 6 | 20 | | 0.2 | 147.23 | 0.4 |
| 18 | | AMAA8805 | L | 6 | 28 | C | 0.2 | 84.11 | 0.4 |
| 19 | | AMAA8805 | T | 6 | 28 | | 0.2 | 163.62 | 0.4 |
| 20* | Griva, 21.12.90 | EDE19001 | L | 6 | 32 | C | 0.2 | 100.13 | 0.57 |
| 21* | (Edessa) | EDE19001 | T | 6 | 32 | | 0.2 | 94.35 | 0.59 |
| 22 | 26.3.93 | PYR19306 | L | 4.9 | 6 | C | 0.25 | 105.64 | 0.21 |
| 23 | | PYR19306 | T | 4.9 | 6 | | 0.25 | 221.48 | 0.18 |
| 24* | Vartholomio, 26.3.93 | PYR19308 | L | 5.4 | 14 | | 0.2 | 162.86 | 0.28 |
| 25* | (Pyrgos) | PYR19308 | T | 5.4 | 14 | | 0.2 | 425.85 | 0.33 |
| 26 | 14.7.93 | PAT19302 | L | 5.6 | 10 | C | 0.2 | 143.74 | 0.3 |
| 27 | | PAT19302 | T | 5.6 | 10 | | 0.2 | 192.49 | 0.36 |
| 28 | | PAT29302 | L | 5.6 | 9 | C | 0.25 | 164.2 | 0.3 |
| 29 | | PAT29302 | T | 5.6 | 9 | | 0.25 | 388.57 | 0.22 |
| 30* | Mt.Vourinos, 13.5.95 | KOZ19501 | L | 6.6 | 16 | A | 0.25 | 211.73 | 0.27 |
| 31* | (Kozani) | KOZ19501 | T | 6.6 | 16 | | 0.25 | 137.38 | 0.26 |
| 32 | Kozani, 15.5.95 | CHR19513 | L | 5.1 | 13 | B | 0.15 | 156.98 | 0.16 |
| 33 | | CHR19513 | T | 5.1 | 13 | | 0.15 | 132.13 | 0.16 |
| 34 | Kozani, 17.5.95 | CHR19532 | L | 5.3 | 11 | | 0.1 | 116.66 | 0.25 |
| 35 | | CHR19532 | T | 5.3 | 11 | | 0.1 | 130.35 | 0.26 |
| 36 | Karpero, 19.5.95 | KRR19501 | L | 5.1 | 12 | B | 0.15 | 185.27 | 0.41 |
| 37 | (Kozani) | KRR19501 | T | 5.1 | 12 | | 0.15 | 261.98 | 0.36 |
| 38 | Karpero 11.6.95 | KRR19509 | L | 4.8 | 5 | | 0.15 | 119.35 | 0.26 |
| 39 | (Kozani) | KRR19509 | T | 4.8 | 5 | | 0.15 | 82.84 | 0.28 |
| 40 | | KEN19563 | L | 4.8 | 7 | C | 0.1 | 125.09 | 0.3 |
| 41 | | KEN19563 | T | 4.8 | 7 | | 0.1 | 100.04 | 0.3 |
| 42 | Konitsa, 5.8.96 | KON29601 | L | 5.7 | 8 | C | 0.1 | 383.68 | 0.38 |
| 43 | | KON29601 | T | 5.7 | 8 | | 0.1 | 381.25 | 0.36 |
| 44 | | KON19601 | T | 5.7 | 8 | B | 0.1 | 168.38 | 0.62 |
| 45 | Strofades, 18.11.97 | ZAK19703 | L | 6.6 | 48 | C | 0.15 | 114.9 | 0.46 |
| 46 | | ZAK19703 | T | 6.6 | 48 | | 0.15 | 129.44 | 0.5 |
| 47 | | ATH39901 | L | 5.9 | 15 | B | 0.2 | 258.59 | 0.34 |
| 48 | | ATH39901 | T | 5.9 | 15 | | 0.2 | 297.19 | 0.27 |
| 49 | | ATH49901 | L | 5.9 | 17 | A | 0.2 | 118.57 | 0.37 |
| 50 | | ATH49901 | T | 5.9 | 17 | | 0.2 | 107.88 | 0.51 |
| 51 | | RFNA9901 | L | 5.9 | 36 | B | 0.25 | 87.45 | 0.28 |
| 52 | | RFNA9901 | T | 5.9 | 36 | | 0.25 | 100.2 | 0.34 |
| 53 | (Sepolia) | SPLB9901 | L | 5.9 | 14 | B | 0.1 | 318.29 | 0.27 |
| 54 | | SPLB9901 | T | 5.9 | 14 | | 0.1 | 306.32 | 0.29 |
| 55 | 14.8.03 | LEF10301 | L | 6.2 | 12 | C | 0.1 | 333.4 | 0.28 |
| 56 | (Lefkada) | LEF10301 | T | 6.2 | 12 | | 0.1 | 408.6 | 0.24 |
| 57 | | PRE10302 | L | 6.2 | 24 | N/A | 0.1 | 153.2 | 0.25 |
| 58 | | PRE10302 | T | 6.2 | 24 | N/A | 0.1 | 141.5 | 0.3 |
| 59 | Kithira, 08.01.06 | KYT10602 | L | 6.9 | 40 | A | 0.07 | 120.4 | 0.47 |
| 60 | | KYT10602 | T | 6.9 | 40 | | 0.07 | 104.1 | 0.47 |
| 61 | | ANS10601 | L | 6.9 | 41 | B | 0.1 | 143.4 | 0.27 |
| 62 | | ANS10601 | T | 6.9 | 41 | | 0.1 | 65.4 | 0.18 |
| 63* | N.Aegean 26.8.83 | POL18302 | L | 5.1 | 47 | A | 0.5 | 90.76 | 0.16 |
| 64* | Aigion, 15.6.95 | AIG95 | L | 6 | 15 | B | N/A | 492 | 0.49 |
| 65* | | AIG95 | T | 6 | 15 | | N/A | 533 | 0.47 |

used which is identical with the one used in the study of Kappos and Kyriakakis [9]. These 21 records are denoted in Table 1 with an asterisk (first column of Table 1) and they permit direct comparison of the results of this study with those of Kappos and

Kyriakakis. A few records of the second set have been filtered with cut-off frequencies higher than 0.25 Hz, however these records are used here only for a comparison with the pertinent results of Kappos and Kyriakakis in terms of coefficient of variation.

4. Implementation of dimensional analysis

In order to assess the proposed dimensional response analysis approach, two fundamental mechanical configurations in earthquake engineering are considered: the elastic–plastic system and the pounding oscillator.

4.1. Elastic–plastic system subjected to base excitation without distinct pulse

The response of an elastic–plastic system subjected to base excitation depends solely on: the specific strength, Q/m and the yield displacement, u_y , of the system, as well as the characteristics of the excitation [1]. By analogy to the case of typical pulses [1], the excitation characteristics are determined by the shape of the accelerogram, the intensity (*PGA*) and the mean period, T_m [19]. While the mean period T_m offers a time scale of the excitation, the product $\alpha_g T_m^2$ (α_g is the *PGA*) corresponds to a length scale which is intimately related with the persistence of the excitation to produce inelastic action. Thus, the length scale is the peak ground acceleration α_g , and the time scale the angular frequency: $\omega_g = 2\pi/T_m$, where T_m is the mean period [19]; accordingly

$$u_{\max} = f(Q/m, u_y, \alpha_g, \omega_g) \quad (2)$$

The five variables appearing in Eq. (2) involve only two reference dimensions, those of length [L] and time [T]. According to Buckingham’s “ Π ” theorem the number of independent dimensionless Π -products is: (5 variables)–(2 reference dimensions) = 3 Π -terms. We choose to normalize the non-linear response, to the energetic length scale of the excitation, $L_e = \alpha_g/\omega_g^2 = (1/4\pi^2) \alpha_g T_m^2$, where α_g is the *PGA*. The main difference with the case of typical pulses [1] is that instead of using the period of the distinguishable pulse, T_p , in the case of excitations with arbitrary shape, we use the mean period of the excitation, T_m . Accordingly, Eq. (2) reduces to

$$\Pi_{\text{um}} = \phi(\Pi_{\text{uy}}, \Pi_Q) \Leftrightarrow \frac{u_{\max}\omega_g^2}{a_g} = \phi\left(\frac{u_y\omega_g^2}{a_g}, \frac{Q}{ma_g}\right) \quad (3)$$

The most critical feature of the proposed approach is that it brings forward the property of self-similarity for excitations of arbitrary shape. Fig. 6, illustrates the self-similar response spectra for a given dimensionless yield displacement, Π_{uy} , and different excitation intensities (Fig. 6 left), which collapse to a single master curve, when expressed in the dimensionless Π -terms (Fig. 6 bottom). In this case (Fig. 6) the excitation is the SPLB9901 record (Table 1), used with different amplitude scaling factor: 0.5/1.0/1.5.

Fig. 7 (left) plots the self-similar response spectra of the elastic–plastic system, for all the selected ground motions of Table 1 and for different dimensionless yield displacements $\Pi_{\text{uy}} = 1.0$ (top), 0.5 (middle) and 0.2 (bottom). Also, in Fig. 7 (right) the corresponding coefficient of variation (*COV*) is illustrated. Hence, the proposed approach can also be evaluated in terms of reducing the scatter in the response. It is shown that the *COV* remains in most cases below 0.4, which is reasonably low (in an earthquake engineering context). As a general trend the *COVs* increase as the dimensionless yield displacements decrease, but in all cases the *COVs* range (Fig. 7 right) between 0.2 and 0.4.

Fig. 8 presents the same plots as Fig. 7 for the second set of records, which is identical with the one used by Kappos and Kyriakakis [9]. Note that when compared with the *COV* estimated in the study of Kappos and Kyriakakis the proposed approach yields equal or even smaller scatter than the optimal of Kappos and Kyriakakis. It is recalled here that the study of Kappos and Kyriakakis concerned the same mechanical configuration and

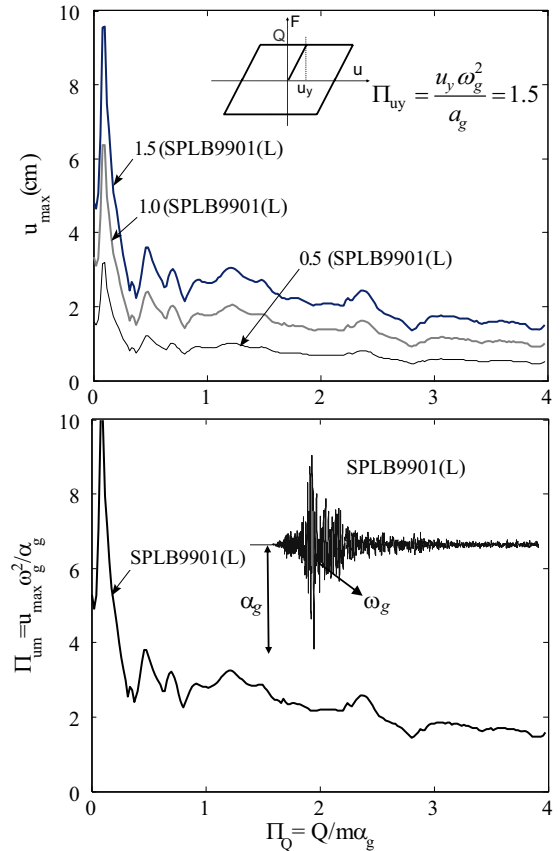


Fig. 6. Top: self-similar response spectra of the elastic–plastic system for the SPLB9901 record (Table 1). Bottom: When the response is presented in the proposed dimensionless terms, all response spectra collapse to a single master curve (self-similarity).

excitations, but it differed in the concept of describing the behaviour, which was based on ductility in lieu of the dimensionless Π -terms proposed herein. This observation underlines the ability of the dimensional analysis approach to scale efficiently the inelastic response of structures.

4.2. Pounding oscillator subjected to base excitation of arbitrary shape

With reference to Fig. 9 and following exactly the same procedure as for the elastic–plastic system, the behaviour of the pounding oscillator can be described using the following dimensionless Π -terms (for more details see [6]):

$$\Pi_{\text{um}} = \phi(\Pi_{\omega_0}, \Pi_{\delta}, \Pi_{\varepsilon}, \Pi_{\xi}) \Leftrightarrow \frac{u_{\max}\omega_g^2}{a_g} = \phi\left(\frac{\omega_0}{\omega_g}, \frac{\delta\omega_g^2}{a_g}, \varepsilon_N, \xi_0\right) \quad (4)$$

where $\omega_0 = 2\pi/T_0$ the angular frequency and ξ_0 the damping ratio of the oscillator, ε_N the coefficient of restitution, δ the size of the gap, and the two parameters which describe the excitation: the length scale, $\alpha_g = \text{PGA}$, and time scale, $\omega_g = 2\pi/T_m$ (T_m is the mean period, see Eq. (1)).

Again, the response spectra of the pounding oscillator (Fig. 9 left) for a given dimensionless gap, Π_{δ} , become self-similar and when expressed in the proposed dimensionless terms, collapse to a single master curve. It is reminded that in the present study, no reference has been made to typical pulses (unlike in [6,7]). The response under ground motions with arbitrary shape has been

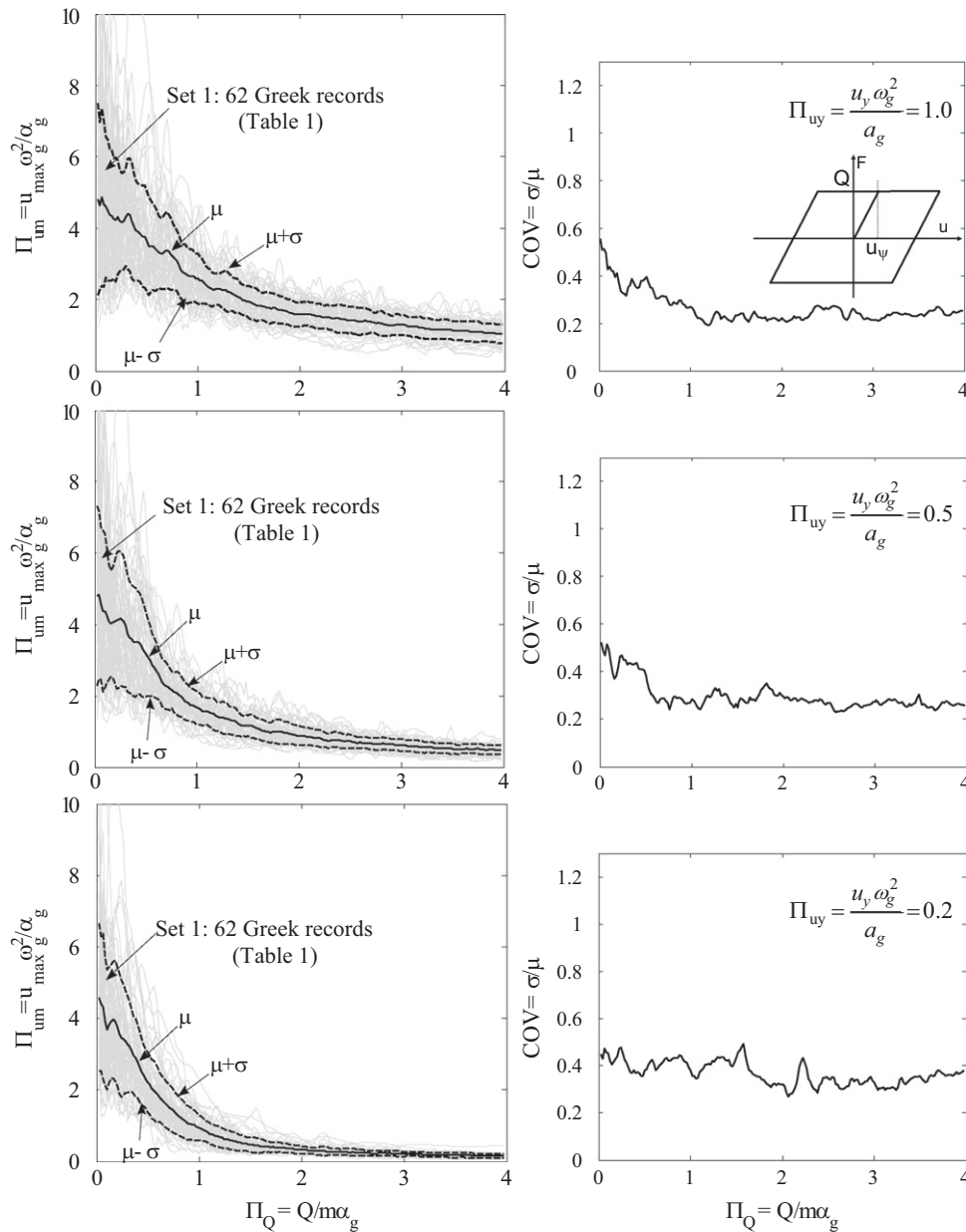


Fig. 7. Left: self-similar response spectra of the elastic–plastic system for the selected excitations (Table 1). (μ): mean value, (σ): standard deviation. Right: coefficient of variation (COV). Dimensionless yield displacement = 1.0 (top), 0.5 (middle), 0.2 (bottom).

scaled directly to the excitation. Furthermore, the proposed approach yields also considerably low scatter in the response, since the coefficient of variation ranges between 0.2 and 0.4 (Fig. 9 right).

5. Conclusions

The aim of the present study was to build on the dimensional response analysis approach introduced in previous works of the authors, and generalize the use of dimensional analysis for describing the response of yielding and pounding structures in the case of excitations without distinct pulses, by proposing the appropriate time and length scales of excitations with arbitrary time history.

The proposed scales were selected among the available in literature strong ground motion parameters, according to predefined criteria. As length scale, the peak ground acceleration (PGA) was adopted, and as time scale the mean period (T_m) [19]. When the response was described in dimensionless terms that hinge upon those parameters (PGA, T_m), remarkable order emerged: response spectra for any excitation level collapsed to a single master curve (self-similarity). Furthermore, it was shown, that this is true for two fundamental mechanical configurations of earthquake engineering, the elastic–plastic system and the pounding oscillator.

The present analysis finally concluded that the proposed approach is also efficient in reducing the scatter in the results and thus, it is a superior method of describing the non-linear structural response of the studied structures, compared with the more ‘conventional’ ones.

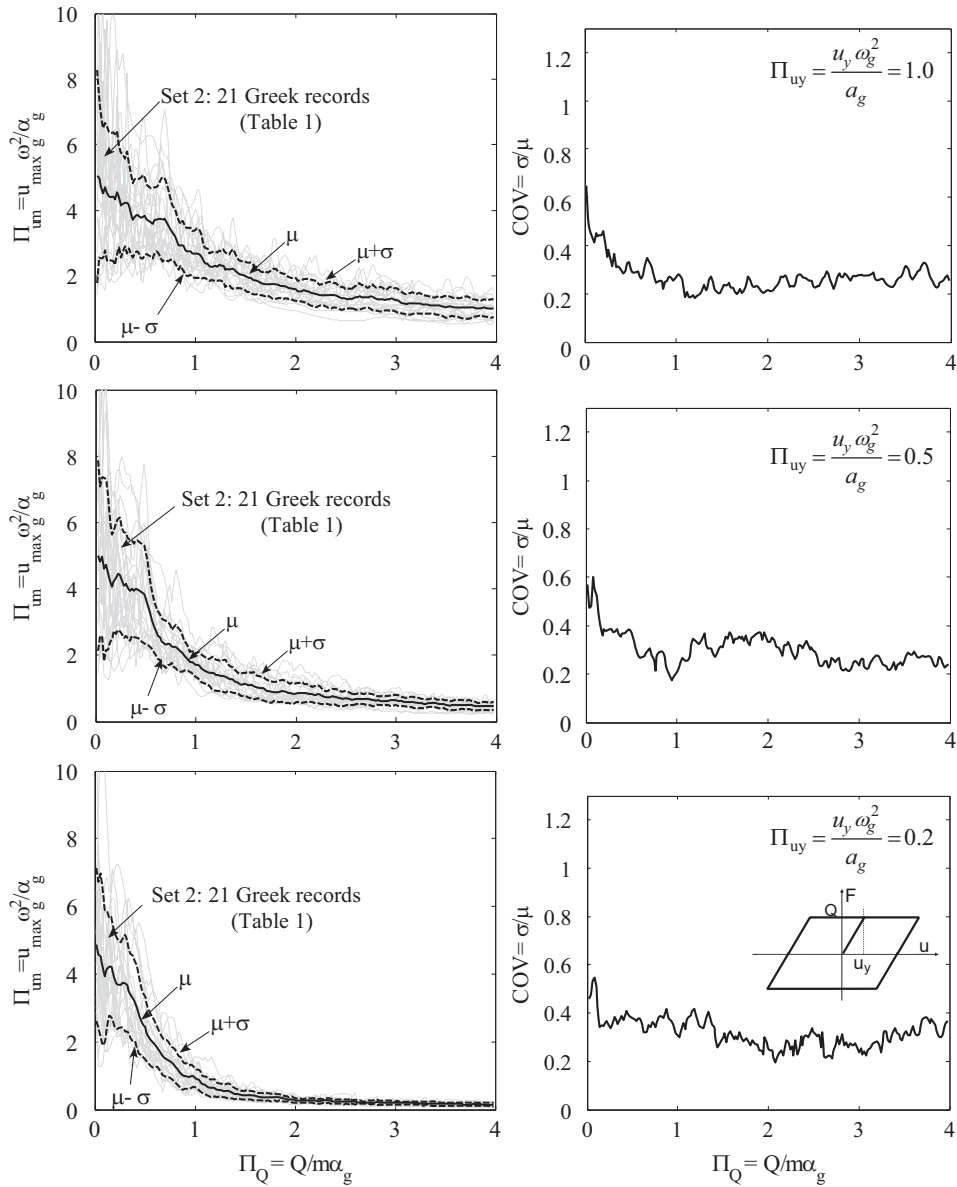


Fig. 8. Left: self-similar response spectra of the elastic-plastic system. Selected excitations (Table 1) for comparison with [9]. (μ): mean value, (σ): standard deviation. Right: coefficient of variation (COV). Dimensionless yield displacement = 1.0 (top), 0.5 (middle), 0.2 (bottom).

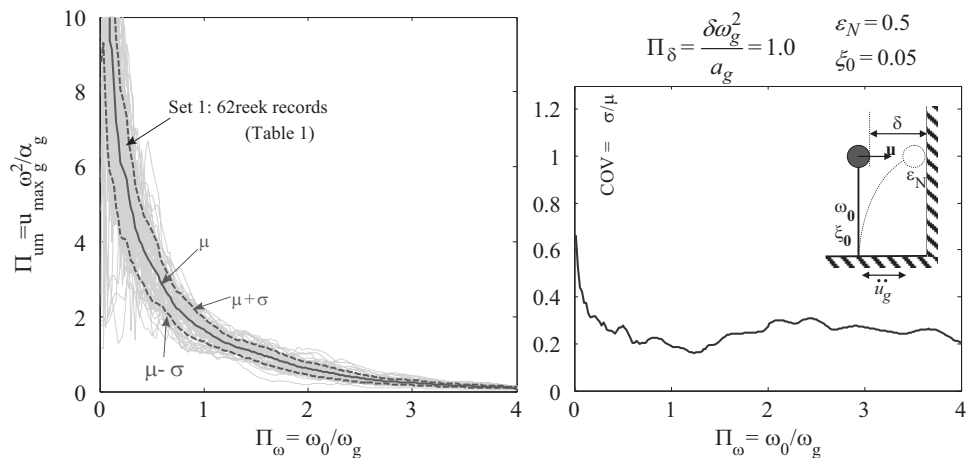


Fig. 9. Left: self-similar response spectra of the pounding oscillator for the selected excitations (Table 1). (μ): mean value, (σ): standard deviation. Right: coefficient of variation (COV).

References

- [1] Makris N, Black CJ. Dimensional analysis of rigid-plastic and elastoplastic structures under pulse-type excitations. *Journal of Engineering Mechanics (ASCE)* 2004;130(9):1006–18.
- [2] Makris N, Black CJ. Dimensional analysis of bilinear oscillators under pulse-type excitations. *Journal of Engineering Mechanics (ASCE)* 2004;130(9):1019–31.
- [3] Barenblatt GI. *Scaling, self-similarity, and intermediate asymptotics*. Cambridge, UK: Cambridge University Press; 1996.
- [4] Sedov LI. *Similarity and dimensional methods of mechanics*. New York: Academic Press; 1959.
- [5] Makris N, Psychogios C. Dimensional response analysis of yielding structures with 1st-mode dominated response. *Earthquake Engineering and Structural Dynamics* 2006;35:1203–24.
- [6] Dimitrakopoulos EG, Makris N, Kappos AJ. Dimensional analysis of pounding oscillators. In: Sixth GRACM international congress on computational mechanics. Thessaloniki 2008, 19–21 June.
- [7] Dimitrakopoulos EG, Makris N, Kappos AJ. Dimensional Analysis of the Earthquake Induced Pounding between adjacent Oscillators. *Earthquake Engineering & Structural Dynamics* (in press published on-line) 2008.
- [8] Mavroeidis GP, Papageorgiou AS. A mathematical representation of near-fault ground motions. *Bulletin of the Seismological Society of America* 2003;93(3):1099–131.
- [9] Kappos AJ, Kyriakakis P. A re-evaluation of scaling techniques for natural records. *Soil Dynamics and Earthquake Engineering* 2000;20:111–23.
- [10] Alavi B, Krawinkler H. Effects of near-source ground motions on frame-structures The John A. Blume Earthquake Engineering Center, Stanford University, 2001
- [11] Makris N, Chang S. Effect of viscous, visco-plastic and friction damping on the response of seismic isolated structures. *Earthquake Engineering Structural Dynamics* 2000;29:85–107.
- [12] Mavroeidis GP, Dong G, Papageorgiou AS. Near-fault ground motions, and the response of elastic and inelastic single-degree-of-freedom (SDOF) systems. *Earthquake Engineering Structural Dynamics* 2004;33:1023–49.
- [13] Bray JD, Rodriguez-Marek A. Characterization of forward-directivity ground motions in the near-fault region. *Soil Dynamics and Earthquake Engineering* 2004;24:815–28.
- [14] Aldea A, Lungu D, Arion A. GIS mapping of seismic microzonation and Site effects in Bucharest Based on Existing seismic and Geophysical Evidence. In: Proceedings of the International Conference "Earthquake Loss Estimation and Risk Reduction" Bucharest, Ro. 2004, October 24–26, 2002, p. 237–50.
- [15] Hancock J, Bommer JJ. A state-of-knowledge review of the influence of strong-motion duration on structural damage. *Earthquake Spectra* 2006;22(3):827–45.
- [16] Hancock J, Bommer JJ. Using spectral matched records to explore the influence of strong-motion duration on inelastic structural response. *Soil Dynamics and Earthquake Engineering* 2007;27:291–9.
- [17] Kappos AJ. Sensitivity of calculated inelastic seismic response to input motion characteristics. In: Proceedings of the Fourth US national conference on earthquake engineering. Oakland California, 1990, p. 25–34.
- [18] Mylonakis G, Voyagaki E. Yielding oscillator subjected to simple pulse waveforms: numerical analysis & closed-form solutions. *Earthquake Engineering Structural Dynamics* 2006;35:1949–74.
- [19] Rathje EM, Abrahamson NA, Bray JD. Simplified frequency content estimates of earthquake ground motions. *Journal of Geotechnical and Geoenvironmental Engineering* 1998;124(2):150–9.
- [20] Mylonakis G, Reinhorn AM. Yielding oscillator under triangular ground acceleration pulse. *Journal of Earthquake Engineering* 2001;5(2):225–51.
- [21] Housner GW. Intensity of Ground Motion During Strong Earthquakes Second Technical Report, Project Designation NR-081-095 CALTECH, Earthquake Research Laboratory, 1952
- [22] Kappos AJ. Analytical prediction of the collapse earthquake for R/C buildings: suggested methodology. *Earthquake Engineering Structural Dynamics* 1991;20(2):167–76.
- [23] Dimova S, Elenas A. Seismic intensity parameters for fragility analysis of structures with energy dissipating devices. *Structural Safety* 2002;24:1–28.
- [24] Fajfar P, Vidic T, Fischinger M. A measure of earthquake motion capacity to damage medium-period structures. *Soil Dynamics and Earthquake Engineering* 1990;9(5):236–42.
- [25] Riddell R. On ground motion intensity indices. *Earthquake Spectra* 2007;23(1):147–73.
- [26] Makris N, Black CJ. Evaluation of peak ground velocity as a "Good" intensity measure for near-source ground motions. *Journal of Engineering Mechanics (ASCE)* 2004;130(9):1032–44.
- [27] Cabanas L, Benito B, Herraiz M. An approach to the measurement of the potential structural damage of earthquake ground motions. *Earthquake Engineering Structural Dynamics* 1997;26:79–92.
- [28] Cosenza E, Manfredi G. Damage indices and damage measures. *Progress in Structural Engineering and Materials* 2000;2:50–9.
- [29] Manfredi G, Polese M, Cosenza E. Cumulative demand of the earthquake ground motions in the near source. *Earthquake Engineering Structural Dynamics* 2003;32:1853–65.
- [30] Karakostas CZ, Athanassiadou CJ, Kappos AJ, Lekidis VA. Site-dependent design spectra and strength modification factors, based on records from Greece. *Soil Dynamics and Earthquake Engineering* 2007;27:1012–27.